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Non-singular inflation with vacuum decay

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Abstract

On the basis of a semi-classical analysis of vacuum energy in an expanding spacetime, we describe a non-singular cosmological model in which the vacuum density decays with time, with a concomitant production of matter. During an infinitely long period we have an empty, inflationary universe, with $H \approx 1$. This primordial era ends in a fast phase transition, during which H and Λ decrease to nearly zero in a few Planck times, with the release of a huge amount of radiation. The late-time scenario is similar to the standard model, with the radiation phase followed by a long dust era, which tends asymptotically to a de Sitter universe, with vacuum dominating again. An analysis of the redshift-distance relation for supernovae Ia leads to cosmological parameters in agreement with other current estimations.

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1. Introduction

The role of vacuum energy in cosmology has acquired a renewed importance, with current observations pointing to the existence of a negative-pressure component in the cosmic fluid. This has reinforced the cosmological constant problem [1], whose origin can be understood if we write the energy density associated with vacuum quantum fluctuations. In the case of a massless scalar field, it is given by the divergent integral

$$\Lambda_0 \approx \int_0^\infty \omega^3 d\omega. \quad (1)$$

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This integral can be regularized by imposing a cutoff m in the superior limit of integration, leading to $\Lambda_0 \approx m^4$. The same result can be derived by introducing a bosonic distribution function in (1),

$$\Lambda_0 \approx \int_0^\infty \frac{\omega^3 d\omega}{e^{\omega/m} - 1} \approx m^4, \quad (2)$$

which is equivalent to consider the vacuum fluctuations thermally distributed, at a characteristic energy m .

For a cutoff of the order of the Planck mass, this leads to a vacuum density of 120 orders of magnitude above the currently observed dark energy density. One may argue that m should in fact be much smaller, because vacuum fluctuations above the energy scale of the QCD phase transition—the latest cosmological vacuum transition—would lead to quark deconfinement. However, even with this value for m , we obtain a vacuum density of 40 orders of magnitude above the observed value.

The cosmological constant problem may be alleviated by the following reasoning. The above energy density is obtained in a flat spacetime; but in such a background the energy–momentum tensor in Einstein equations should be zero. Therefore, by consistence, the above vacuum density must be exactly cancelled by a bare cosmological constant. If we now derive the vacuum contribution in a curved spacetime, we should expect, after the subtraction of the bare cosmological constant, a renormalized, curvature-dependent vacuum energy density. For instance, in the case of an expanding, spatially isotropic and homogeneous spacetime, filled with vacuum and matter components, the renormalized Λ should be time dependent, being very high at early times, but decreasing to zero or nearly zero as the universe expands [2].

To have an idea of what a varying cosmological term may be, let us initially consider the case of a de Sitter spacetime. It is generally believed that the de Sitter cosmological horizon has an associated temperature given by $H/2\pi$, where $H = \dot{a}/a$ is the Hubble parameter [3]². Therefore, a phenomenological expression for the effective vacuum density in this case may be derived by substituting $m + H$ for the energy scale m in (2), with a subsequent subtraction of $\Lambda_0 \approx m^4$. In this way we obtain

$$\Lambda \approx (m + H)^4 - m^4. \quad (3)$$

In the limit $H \gg m$, this leads to the cutoff-independent result $\Lambda \approx H^4$. Since, in a de Sitter spacetime, $\Lambda \approx H^2$, this implies that $H \approx 1$, that is, this limit necessarily describes a de Sitter universe with a horizon radius of the order of the Planck length. On the other hand, in the limit $H \ll m$, we have $\Lambda \approx m^3 H$. Using again $\Lambda \approx H^2$, we obtain $H \approx m^3$, or $\Lambda \approx m^6$. If, as discussed above, we choose m as the energy scale of the QCD vacuum phase transition (of the order of the pion mass), the first result is an expression of Dirac's large number coincidence [7], while the last one gives approximately the current observed value of Λ [8].

The above discussion concerns stationary spacetimes, for which H and Λ are truly constants. Nevertheless, it suggests the possibility of a universe evolving from an initial, asymptotically de Sitter phase, with $\Lambda \sim 1$, to a final, asymptotically de Sitter phase with $\Lambda \sim m^6 \ll 1$, with Λ decreasing with time according to (3). Before verifying such a possibility in the following sections, let us briefly discuss the energy conservation in this context.

There is a common belief about the impossibility of a varying cosmological term, because of the Bianchi identities and the covariant conservation of the energy–momentum of matter.

² This result was derived by Gibbons and Hawking on the basis of Euclidian methods [4], but the positiveness of the de Sitter temperature depends on some appropriate physical interpretation (see, for example, [5, 6]).

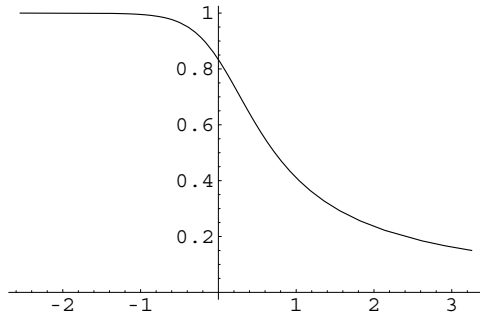


Figure 1. The Hubble parameter as a function of time (in Planck units).

Indeed, the Bianchi identities $G^\mu_{\nu;\mu} = 0$ (G is the Einstein tensor) imply, via Einstein’s equations, the conservation of the total energy–momentum tensor, $T^\mu_{\nu;\mu} = \tilde{T}^\mu_{\nu;\mu} + g^\mu_\nu \Lambda_{,\mu} = 0$, where \tilde{T} is the energy–momentum tensor of matter. If one assumes the independent conservation of matter, i.e., $\tilde{T}^\mu_{\nu;\mu} = 0$, it follows that $\Lambda_{,\mu} = 0$, that is, Λ is a constant.

However, if matter is not independently conserved, Λ may vary with time. In the realm of a FRW spacetime, the conservation of the total energy–momentum tensor reduces to the form

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \tag{4}$$

where ρ_T and p_T are the total energy density and pressure, respectively. By using $\rho_T = \rho_m + \Lambda$ and $p_T = p_m - \Lambda$ (ρ_m and p_m refer to the corresponding matter quantities), we have

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -\dot{\Lambda}. \tag{5}$$

In other words, the vacuum decay is concomitant with a process of matter production, in order to preserve the covariant conservation of the total energy.

2. The early times

In the spatially flat case, the Friedmann equations give $\rho_T = 3H^2$. In the limit of very early times we can take $H \gg m$, and (3) reduces to $\Lambda = 3H^4$, where the factor 3 is not important and was chosen for mathematical convenience. Using for matter the equation of state of radiation, $p_m = \rho_m/3$, the conservation equation (5) then leads to the evolution equation:

$$\dot{H} + 2H^2 - 2H^4 = 0. \tag{6}$$

For $0 < H < 1$, the solution of (6) is given by

$$2t = \frac{1}{H} - \tanh^{-1} H, \tag{7}$$

where t is the cosmological time, and an integration constant was conveniently chosen.

This solution is plotted in figure 1, with t and H expressed in Planck units. We can see that this universe has no initial singularity, existing since an infinite past, when it approaches asymptotically a de Sitter state with $H = 1$. During an infinitely long period we have a quasi-de Sitter universe, with $H \approx \Lambda \approx 1$. However, at a given time (arbitrarily chosen around $t = 0$), the expansion undertakes a fast and huge phase transition, with H and Λ decreasing to nearly zero in a few Planck times³.

³ The time at which the transition occurs depends on the integration constant in (7). Note, however, that it takes place at a definite time before the present time. In this sense, we can attribute an age for the subsequent universe, which will be determined below.

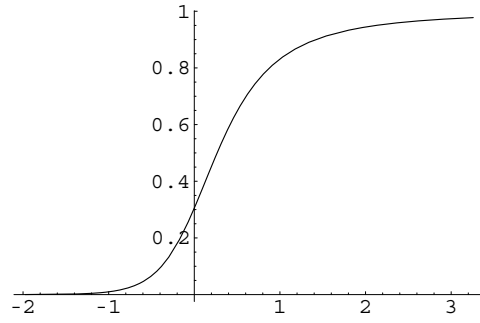


Figure 2. The relative energy density of radiation as a function of time.

From $\rho_m = \rho_r - \Lambda$, we can obtain $\Omega_m = 1 - H^2$, where $\Omega_m = \rho_m/3H^2$ is the relative density of matter. Its time variation is plotted in figure 2. One can see that Ω_m changes suddenly during the phase transition, from nearly zero, in the quasi-de Sitter phase, to nearly 1 at the end of the transition. In this way, the present solution has some attributes of an inflationary universe, with an infinitely long period of inflation ending in a fast transition during which the vacuum decays, releasing a huge amount of energy in the form of radiation and relativistic matter. After the transition we have a radiation-dominated FRW universe, whose subsequent evolution, as we will see, is similar to the standard Λ CDM recipe.

3. Late times

In the opposite limit we can take $H \ll m$, and (3) reduces to $\Lambda \approx m^3 H$. Let us write it as $\Lambda = \sigma H$, with $\sigma \approx m^3$, and let us introduce the equation of state of matter, $p_m = (\gamma - 1)\rho_m$. From (5) we now have

$$2\dot{H} + 3\gamma H^2 - \sigma\gamma H = 0. \quad (8)$$

For $H > 0$ and $\rho_m > 0$, one has the solution [9]

$$a(t) = C[\exp(\sigma\gamma t/2) - 1]^{\frac{2}{3\gamma}}, \quad (9)$$

where C is an integration constant, and a second one was conveniently chosen.

From it we can derive $H(a)$ and, with the help of $\rho_m = 3H^2 - \Lambda$, we obtain

$$\rho_m = \frac{\sigma^2}{3} \left(\frac{C}{a}\right)^{3\gamma/2} \left[1 + \left(\frac{C}{a}\right)^{3\gamma/2}\right], \quad (10)$$

$$\Lambda = \frac{\sigma^2}{3} \left[1 + \left(\frac{C}{a}\right)^{3\gamma/2}\right]. \quad (11)$$

3.1. The radiation era

In the radiation phase, doing $\gamma = 4/3$ and taking the limit $\sigma t \ll 1$, we have

$$a \approx \sqrt{2C^2\sigma t/3}, \quad (12)$$

$$\rho_m = \frac{\sigma^2 C^4}{3a^4} = \frac{3}{4t^2}, \tag{13}$$

and

$$\Lambda = \frac{\sigma^2 C^2}{3a^2} = \frac{\sigma}{2t}. \tag{14}$$

The two first results are the same we obtain in the standard model. The third one shows that, in this limit, Λ is sub-dominant compared to ρ_m , and, therefore, the matter production can be neglected. This is the reason why radiation is conserved, with an energy density scaling with a^{-4} .

Equations (12) and (13) guarantee that physical processes taking place during the radiation phase are not affected by the vacuum decay. For example, the primordial nucleosynthesis remains unchanged, since the expansion and reaction rates are the same as in the standard context.

3.2. The matter era

In the case of a matter fluid dominated by dust, taking $\gamma = 1$ we obtain, from (9),

$$a(t) = C[\exp(\sigma t/2) - 1]^{2/3}. \tag{15}$$

In the limit $\sigma t \ll 1$, it reduces to

$$a(t) = C(\sigma t/2)^{2/3}, \tag{16}$$

that is, the scale factor evolves as in a dust-dominated Einstein–de Sitter universe.

On the other hand, for $t \rightarrow \infty$ we have

$$a(t) = C \exp(\sigma t/3), \tag{17}$$

i.e., our solution tends asymptotically to a de Sitter universe, with $H = \sigma/3$.

For the matter and vacuum energy densities, we obtain, from (10) and (11),

$$\rho_m = \frac{\sigma^2 C^3}{3a^3} + \frac{\sigma^2 C^{3/2}}{3a^{3/2}}, \tag{18}$$

$$\Lambda = \frac{\sigma^2}{3} + \frac{\sigma^2 C^{3/2}}{3a^{3/2}}. \tag{19}$$

It is not difficult to interpret these results. The first terms in these equations are the standard expressions for the scaling of matter and the cosmological term, which are valid if there is no matter production. The first term of (18) is dominant in the limit $a/C \ll 1$, when a scales as in (16). The first term of (19), on the other hand, is dominant for $a \rightarrow \infty$, acting as a genuine cosmological constant.

The second terms in (18) and (19) are related to the vacuum decay. For large times, the matter density decreases slower than in the Λ CDM model, because of the matter production.

3.3. Supernova constraints

As we have seen, the cosmological scenario presented here is similar, on a qualitative level, to the standard scenario of cosmic evolution. Nevertheless, the vacuum decay constitutes a substantial difference at late times, and we should verify its consequences for the quantitative determination of cosmological parameters like the relative density of matter and the universe

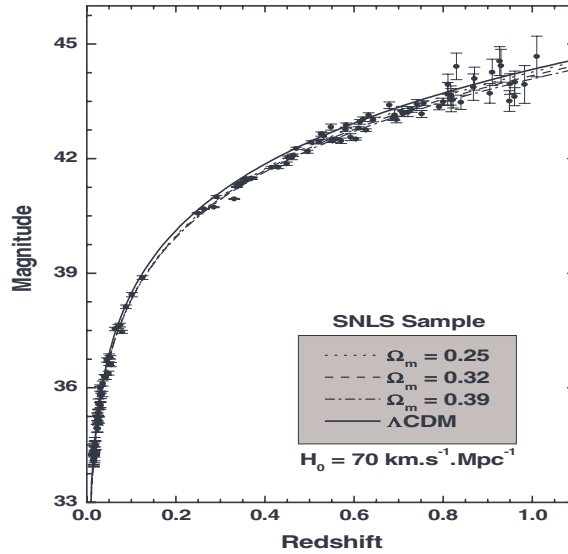


Figure 3. Hubble diagram for 115 supernovae from SNLS Collaboration [10]. The curves correspond to $H_0 = 70 \text{ Km s}^{-1} \text{ Mpc}^{-1}$, and selected values of Ω_m . For the sake of comparison, the flat ΛCDM scenario with $\Omega_m = 0.27$ is also shown.

age, for example. For this purpose, the analysis of the redshift-distance relation for supernovae Ia is of particular importance [10].

From (15), it is easy to derive the Hubble parameter as a function of the redshift $z = a_0/a - 1$, where a_0 is the present value of the scale factor. One obtains

$$H(z) = H_0[1 - \Omega_m + \Omega_m(1+z)^{3/2}], \quad (20)$$

where H_0 and Ω_m are the present values of the Hubble parameter and of the relative density of matter, respectively.

One can see that, as in the ΛCDM model, we have two parameters to be adjusted by fitting the supernova data. In order to do it, we have used (20) to fit the data of the Supernova Legacy Survey (SNLS) Collaboration [11]. The result is shown in figure 3, where we have plotted our theoretical redshift-distance relation for three different values of Ω_m , together with the theoretical relation predicted by the spatially flat ΛCDM model with $\Omega_m = 0.27$. In all cases we have used $H_0 = 70 \text{ Km s}^{-1} \text{ Mpc}^{-1}$.

The best fit obtained with (20) is given by $\Omega_m = 0.32$ and $H_0 = 70 \text{ Km s}^{-1} \text{ Mpc}^{-1}$, with $\chi_r^2 = 1.0$. With 95% of confidence level, we have $0.27 \leq \Omega_m \leq 0.37$ and $0.68 \leq h \leq 0.72$ ($h \equiv H_0/100 \text{ Km s}^{-1} \text{ Mpc}^{-1}$).

With these results we can estimate the universe age in this model. From (15), we can derive an age parameter given by

$$H_0 t_0 = \frac{\frac{2}{3} \ln(\Omega_m)}{\Omega_m - 1}. \quad (21)$$

By using the obtained values of H_0 and Ω_m , one has $t_0 \simeq 15.7 \text{ Gyr}$, corresponding to an age parameter $H_0 t_0 = 1.12$.

It is also possible to obtain from (15) the present deceleration parameter, whose best value is $q = -0.52$. The redshift of transition between the decelerated phase and the accelerated one

is $z = 1.62$ [10], showing that we have a decelerated phase long enough to permit structure formation.

4. Conclusions

We have described, on the basis of a macroscopic approach to vacuum dynamics, a non-singular cosmological scenario in agreement with our general standard view about the universe evolution. We have an initially empty, inflationary spacetime, which is driven to a radiation-dominated phase through a fast phase transition during which a huge amount of energy is released at the expenses of the vacuum decay. The radiation phase is indistinguishable from the standard one, and it is followed by a matter-dominated era, which evolves to a final de Sitter phase, with vacuum dominating again.

This scenario is also consistent with a quantitative analysis of the observed Hubble diagram for supernovae of high redshifts, leading to cosmological parameters in accordance with other—non-cosmological—estimations. In particular, the matter density and age parameters are in the intervals imposed, respectively, by dynamical estimations of dark matter [12] and globular clusters observations [13].

The reader may object that our matter density parameter is above the values estimated on the basis of current observations of the cosmic background radiation [14] and barion acoustic oscillations [15]—despite the superposition of the intervals of allowed values. Nevertheless, it must be emphasized that such estimations are dependent on the adopted cosmological model, and should be redone in our case. In particular, the production of matter modifies the standard relations between the dynamic parameters at the time of last scattering and the present matter density. An analysis of such observations in the context of the present model is in progress.

Another point to be analysed is the evolution of density perturbations, which could be modified by the matter production. A preliminary investigation has shown no important difference in the growing of the density contrast in the matter era, while in the radiation phase the matter production is irrelevant, as discussed above.

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